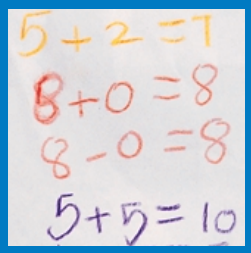
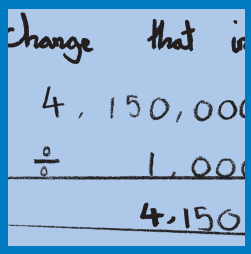
Mathematics





The National Curriculum for England www.nc.uk.net

Excellence in schools

About mathematics in the National Curriculum

The structure of the National Curriculum

The programmes of study[1] set out what pupils should be taught, and the attainment targets set out the expected standards of pupils' performance. It is for schools to choose how they organise their school curriculum to include the programmes of study for mathematics.

The programmes of study

The programmes of study set out what pupils should be taught in mathematics at key stages 1, 2, 3 and 4 and provide the basis for planning schemes of work. When planning, schools should also consider the general teaching requirements for inclusion, use of language and use of information and communication technology that apply across the programmes of study.

The **Knowledge, skills and understanding** in the programmes of study identify the main aspects of mathematics in which pupils make progress:

At key stage 1
- number
- shape, space and measures

At key stage 2
- number
- shape, space and measures
- handling data

At key stages 3 and 4
- number and algebra
- shape, space and measures
- handling data.

There are requirements for using and applying mathematics in each of these sections.

At key stage 1, teaching should ensure that appropriate connections are made between the sections on number, and shape, space and measures.

At key stage 2, teaching should ensure that appropriate connections are made between the sections on number; shape, space and measures; and handling data.

At key stages 3 and 4, teaching should ensure that appropriate connections are made between the sections on number and algebra; shape, space and measures; and handling data.

These aspects are developed through a range of practical activities using mathematical ideas as set out in **Breadth of study**.

The *Framework for teaching mathematics* provides detailed objectives for planning and teaching mathematics for pupils aged five to 11. Those schools that fully implement the *Framework* will fulfil their statutory duty in relation to the National Curriculum for mathematics at key stages 1 and 2.

[1] The Education Act 1996, section 353b, defines a programme of study as the 'matters, skills and processes' that should be taught to pupils of different abilities and maturities during the key stage.

The programmes of study for mathematics

The importance of mathematics
Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem-solving skills, and the ability to think in abstract ways. Mathematics is important in everyday life, many forms of employment, science and technology, medicine, the economy, the environment and development, and in public decision-making. Different cultures have contributed to the development and application of mathematics. Today, the subject transcends cultural boundaries and its importance is universally recognised. Mathematics is a creative discipline. It can stimulate moments of pleasure and wonder when a pupil solves a problem for the first time, discovers a more elegant solution to that problem, or suddenly sees hidden connections.

The huge number project

How long would a tragic jam be with 1,000,000 cars in it?

First of all we measured Mr Jones's Ford Orion, it came out at 4.15 metres.

4.15
X 1,000,000
4,150,000

We worked out that 1,000,000 Ford Orions would be 4,150,000 m.

We needed to change that into km. So we dinded it by 1,000.

4,150,000
÷ 1,000
4,150 km

Then we hit a problem. We realised that there would be a gap in between the cars of about 1 m.
So that would add on 1,000,000 m

5,150,000
÷ 1000
5,150 km

Then we used an atlas to see what countries the first car would have reached by the time the last one left London.

Maths is the study of patterns abstracted from the world around us – so anything we learn in maths has literally thousands of applications, in arts, sciences, finance, health and leisure!

Professor Ruth Lawrence, University of Michigan

Mathematics is not just a collection of skills, it is a way of thinking. It lies at the core of scientific understanding, and of rational and logical argument.

Dr Colin Sparrow, Lecturer in Mathematics, University of Cambridge

Maths is the truly global language. With it, we convey ideas to each other that words can't handle – and bypass our spoken Tower of Babel.

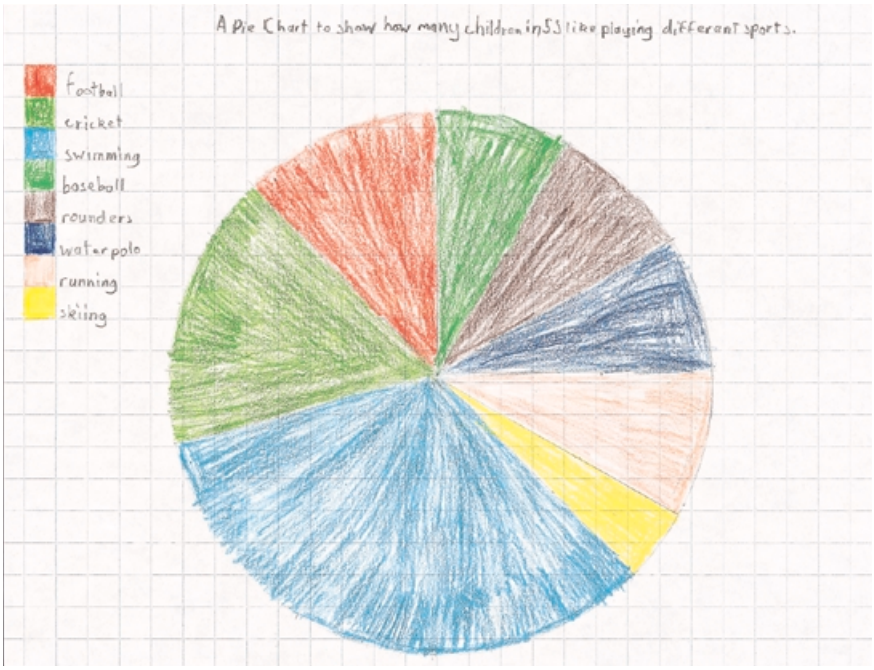
Professor Alison Wolf, Head of Mathematical Sciences Group, Institute of Education, University of London

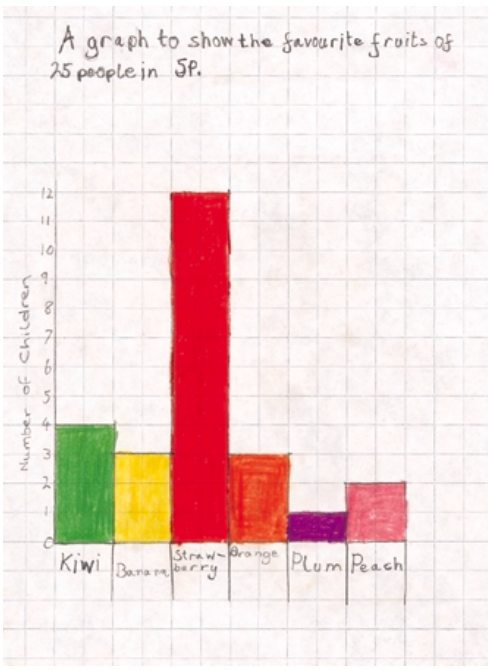
If you want to take part in tomorrow's world, you'll need mathematics and statistics just as much as grammar and syntax.

Professor Robert Worcester, Chairman, Market Opinion Research International

Since the age of ten, I've been hooked on mathematical problems as intellectual challenges. However, nobody has to worry that pure mathematics won't be used. Mathematics – even some of the most abstruse mathematics that we thought would never be used – is now used every time you use your credit card, every time you use your computer.

Professor Andrew Wiles, Princeton University





Programme of study: mathematics

Key stage 1

During key stage 1 pupils develop their knowledge and understanding of mathematics through practical activity, exploration and discussion. They learn to count, read, write and order numbers to 100 and beyond. They develop a range of mental calculation skills and use these confidently in different settings. They learn about shape and space through practical activity which builds on their understanding of their immediate environment. They begin to grasp mathematical language, using it to talk about their methods and explain their reasoning when solving problems.

The mathematics programmes of study and the National Numeracy Strategy *Framework for teaching mathematics* are fully aligned. The *Framework* provides a detailed basis for implementing the statutory requirements of the programme of study for key stage 1 in mathematics.

Building on the early learning goals
Pupils' prior experience of mathematics includes:

- counting and using numbers to at least 10 in familiar contexts
- recognising numerals 1 to 9
- talking about and creating simple patterns
- beginning to understand addition as combining two groups of objects and subtraction as 'taking away'
- describing the shape and size of solid and flat shapes
- using everyday words to describe position
- using early mathematical ideas to solve practical problems.

Note about sections
There is no separate section of the programme of study numbered Ma1 that corresponds to the first attainment target, **using and applying mathematics**. Teaching requirements relating to this attainment target are included within other sections of the programme of study.

Knowledge, skills and understanding

Teaching should ensure that appropriate connections are made between the sections on **number** and **shape, space and measures**.

Ma2 Number

Using and applying number

1 Pupils should be taught to:

Problem solving

a approach problems involving number, and data presented in a variety of forms, in order to identify what they need to do
b develop flexible approaches to problem solving and look for ways to overcome difficulties
c make decisions about which operations and problem-solving strategies to use
d organise and check their work

Communicating

e use the correct language, symbols and vocabulary associated with number and data
f communicate in spoken, pictorial and written form, at first using informal language and recording, then mathematical language and symbols

Reasoning

g present results in an organised way
h understand a general statement and investigate whether particular cases match it
i explain their methods and reasoning when solving problems involving number and data.

Numbers and the number system

2 Pupils should be taught to:

Counting

a count reliably up to 20 objects at first and recognise that if the objects are rearranged the number stays the same; be familiar with the numbers 11 to 20; gradually extend counting to 100 and beyond

Number patterns and sequences

b create and describe number patterns; explore and record patterns related to addition and subtraction, and then patterns of multiples of 2, 5 and 10 explaining the patterns and using them to make predictions; recognise sequences, including odd and even numbers to 30 then beyond; recognise the relationship between halving and doubling

The number system

c read and write numbers to 20 at first and then to 100 or beyond; understand and use the vocabulary of comparing and ordering these numbers; recognise that the position of a digit gives its value and know what each digit represents, including zero as a place-holder; order a set of one- and two-digit numbers and position them on a number line and hundred-square; round any two-digit number to the nearest 10.

Calculations

3 Pupils should be taught to:

Number operations and the relationships between them

a understand addition and use related vocabulary; recognise that addition can be done in any order; understand subtraction as both 'take away' and 'difference' and use the related vocabulary; recognise that subtraction is the inverse of addition; give the subtraction corresponding to an addition and vice versa; use the symbol '=' to represent equality; solve simple missing number problems [for example, 6 = 2 + □]

b understand multiplication as repeated addition; understand that halving is the inverse of doubling and find one half and one quarter of shapes and small numbers of objects; begin to understand division as grouping (repeated subtraction); use vocabulary associated with multiplication and division

Mental methods

c develop rapid recall of number facts: know addition and subtraction facts to 10 and use these to derive facts with totals to 20, know multiplication facts for the ×2 and ×10 multiplication tables and derive corresponding division facts, know doubles of numbers to 10 and halves of even numbers to 20

d develop a range of mental methods for finding, from known facts, those that they cannot recall, including adding 10 to any single-digit number, then adding and subtracting a multiple of 10 to or from a two-digit number; develop a variety of methods for adding and subtracting, including making use of the facts that addition can be done in any order and that subtraction is the inverse of addition

e carry out simple calculations of the form 40 + 30 = □, 40 + □ = 100, 56 – □ = 10; record calculations in a number sentence, using the symbols +, –, ×, ÷ and = correctly [for example, 7 + 2 = 9].

1e, 1f → links to other subjects
These requirements build on En1/1b–1c and En3/1c.

1f → ICT opportunity
Pupils could use ICT to communicate results using appropriate mathematical symbols.

Note for 1i
Explaining methods is an important foundation for reasoning and proof in later key stages.

Note for 5
This provides a basis for pupils' understanding of handling data in later key stages.

Solving numerical problems

4 Pupils should be taught to:
- a choose sensible calculation methods to solve whole-number problems (including problems involving money or measures), drawing on their understanding of the operations
- b check that their answers are reasonable and explain their methods or reasoning.

Processing, representing and interpreting data

5 Pupils should be taught to:
- a solve a relevant problem by using simple lists, tables and charts to sort, classify and organise information
- b discuss what they have done and explain their results.

Ma3 Shape, space and measures

Using and applying shape, space and measures

1 Pupils should be taught to:

Problem solving

a try different approaches and find ways of overcoming difficulties when solving shape and space problems

b select and use appropriate mathematical equipment when solving problems involving measures or measurement

c select and use appropriate equipment and materials when solving shape and space problems

Communicating

d use the correct language and vocabulary for shape, space and measures

Reasoning

e recognise simple spatial patterns and relationships and make predictions about them

f use mathematical communication and explanation skills.

Understanding patterns and properties of shape

2 Pupils should be taught to:

a describe properties of shapes that they can see or visualise using the related vocabulary

b observe, handle and describe common 2-D and 3-D shapes; name and describe the mathematical features of common 2-D and 3-D shapes, including triangles of various kinds, rectangles including squares, circles, cubes, cuboids, then hexagons, pentagons, cylinders, pyramids, cones and spheres

c create 2-D shapes and 3-D shapes

d recognise reflective symmetry in familiar 2-D shapes and patterns.

Understanding properties of position and movement

3 Pupils should be taught to:

a observe, visualise and describe positions, directions and movements using common words

b recognise movements in a straight line (translations) and rotations, and combine them in simple ways [for example, give instructions to get to the headteacher's office or for rotating a programmable toy]

c recognise right angles.

1b → ICT opportunity
Pupils could use both digital and analogue devices to measure weight or time.

1d → links to other subjects
This requirement builds on En1/1b.

Note for 1f
These skills are important foundations for geometrical reasoning and proof in later key stages.

Note for 4a
In the international system of units, kilogram (kg) is the unit of mass. In practice, mass is measured by weighing; scales measure or compare a force (a push or a pull). At key stage 1 it is acceptable to treat weight as synonymous with mass.

4b → ICT opportunity
Pupils could programme a toy to follow a path involving half- and quarter-turns.

Understanding measures

4 Pupils should be taught to:
- a estimate the size of objects and order them by direct comparison using appropriate language; put familiar events in chronological order; compare and measure objects using uniform non-standard units [for example, a straw, wooden cubes], then with a standard unit of length (cm, m), weight (kg), capacity (l) [for example, 'longer or shorter than a metre rule', 'three-and-a-bit litre jugs']; compare the durations of events using a standard unit of time
- b understand angle as a measure of turn using whole turns, half-turns and quarter-turns
- c estimate, measure and weigh objects; choose and use simple measuring instruments, reading and interpreting numbers, and scales to the nearest labelled division.

Breadth of study

1 During the key stage, pupils should be taught the **Knowledge, skills and understanding** through:
- a practical activity, exploration and discussion
- b using mathematical ideas in practical activities, then recording these using objects, pictures, diagrams, words, numbers and symbols
- c using mental images of numbers and their relationships to support the development of mental calculation strategies
- d estimating, drawing and measuring in a range of practical contexts
- e drawing inferences from data in practical activities
- f exploring and using a variety of resources and materials, including ICT
- g activities that encourage them to make connections between number work and other aspects of their work in mathematics.

Programme of study: mathematics

Key stage 2

Knowledge, skills and understanding

Teaching should ensure that appropriate connections are made between the sections on **number**, **shape, space and measures**, and **handling data**.

Ma2 Number

Using and applying number

1 Pupils should be taught to:

Problem solving

a make connections in mathematics and appreciate the need to use numerical skills and knowledge when solving problems in other parts of the mathematics curriculum

b break down a more complex problem or calculation into simpler steps before attempting a solution; identify the information needed to carry out the tasks

c select and use appropriate mathematical equipment, including ICT

d find different ways of approaching a problem in order to overcome any difficulties

e make mental estimates of the answers to calculations; check results

Communicating

f organise work and refine ways of recording

g use notation diagrams and symbols correctly within a given problem

h present and interpret solutions in the context of the problem

i communicate mathematically, including the use of precise mathematical language

Reasoning

j understand and investigate general statements [for example, 'there are four prime numbers less than 10', 'wrist size is half neck size']

k search for pattern in their results; develop logical thinking and explain their reasoning.

Numbers and the number system

2 Pupils should be taught to:

Counting

a count on and back in tens or hundreds from any two- or three-digit number; recognise and continue number sequences formed by counting on or back in steps of constant size from any integer, extending to negative integers when counting back

During key stage 2 pupils use the number system more confidently. They move from counting reliably to calculating fluently with all four number operations. They always try to tackle a problem with mental methods before using any other approach. Pupils explore features of shape and space and develop their measuring skills in a range of contexts. They discuss and present their methods and reasoning using a wider range of mathematical language, diagrams and charts.

The mathematics programmes of study and the National Numeracy Strategy *Framework for teaching mathematics* are fully aligned. The *Framework* provides a detailed basis for implementing the statutory requirements of the programme of study for key stage 2 in mathematics.

Note about sections

There is no separate section of the programme of study numbered Ma1 that corresponds to the first attainment target, **using and applying mathematics**. Teaching requirements relating to this attainment target are included within the other sections.

1f → links to other subjects

This requirement builds on En3/1a, 1e.

1g → links to other subjects

This requirement builds on En1/1a, 1d.

Number patterns and sequences

b recognise and describe number patterns, including two- and three-digit multiples of 2, 5 or 10, recognising their patterns and using these to make predictions; make general statements, using words to describe a functional relationship, and test these; recognise prime numbers to 20 and square numbers up to 10 × 10; find factor pairs and all the prime factors of any two-digit integer

Integers

c read, write and order whole numbers, recognising that the position of a digit gives its value; use correctly the symbols <, >, =; multiply and divide any integer by 10 or 100 then extend to multiplying and dividing by 1000; round integers to the nearest 10 or 100 and then 1000; order a set of negative integers, explaining methods and reasoning; multiply and divide decimals by 10 or 100

Fractions, percentages and ratio

d understand unit fractions [for example, $\frac{1}{3}$ or $\frac{1}{8}$] then fractions that are several parts of one whole [for example, $\frac{2}{3}$ or $\frac{5}{8}$], locate them on a number line and use them to find fractions of shapes and quantities

e understand simple equivalent fractions and simplify fractions by cancelling common factors; compare and order simple fractions by converting them to fractions with a common denominator, explaining their methods and reasoning

f recognise the equivalence between the decimal and fraction forms of one half, quarters, tenths and hundredths; understand that 'percentage' means the 'number of parts per 100' and that it can be used for comparisons; find percentages of whole number quantities, using a calculator where appropriate

g recognise approximate proportions of a whole and use simple fractions and percentages to describe them, explaining their methods and reasoning

h solve simple problems involving ratio and direct proportion

Decimals

i understand and use decimal notation for tenths and hundredths in context [for example, order amounts of money, round a sum of money to the nearest £, convert a length such as 1.36 metres to centimetres and vice versa]; locate on a number line, and order, a set of numbers or measurements; then recognise thousandths (only in metric measurements)

j round a number with one or two decimal places to the nearest integer or tenth; convert between centimetres and millimetres or metres, then between millimetres and metres, and metres and kilometres, explaining methods and reasoning.

Calculations

3 Pupils should be taught to:

Number operations and the relationships between them

a develop further their understanding of the four number operations and the relationships between them including inverses; use the related vocabulary; choose suitable number operations to solve a given problem, and recognise similar problems to which they apply

b find remainders after division, then express a quotient as a fraction or decimal; round up or down after division, depending on the context

c understand the use of brackets to determine the order of operations; understand why the commutative, associative and distributive laws apply to addition and multiplication and how they can be used to do mental and written calculations more efficiently

Note for 3c
Pupils do not need to know the names of these laws.

Mental methods

d recall all addition and subtraction facts for each number to 20

e work out what they need to add to any two-digit number to make 100, then add or subtract any pair of two-digit whole numbers; handle particular cases of three-digit and four-digit additions and subtractions by using compensation or other methods [for example, 3000 – 1997, 4560 + 998]

f recall multiplication facts to 10 × 10 and use them to derive quickly the corresponding division facts

g double and halve any two-digit number

h multiply and divide, at first in the range 1 to 100 [for example, 27 × 3, 65 ÷ 5], then for particular cases of larger numbers by using factors, distribution or other methods

Note for 3i, 3j
Pupils are expected to use mental methods if the calculations are suitable.

Written methods

i use written methods to add and subtract positive integers less than 1000, then up to 10000, then add and subtract numbers involving decimals; use approximations and other strategies to check that their answers are reasonable

j use written methods for short multiplication and division by a single-digit integer of two-digit then three-digit then four-digit integers, then of numbers with decimals; then use long multiplication, at first for two-digit by two-digit integer calculations, then for three-digit by two-digit calculations; extend division to informal methods of dividing by a two-digit divisor [for example, 64 ÷ 16]; use approximations and other strategies to check that their answers are reasonable

4d → ICT opportunity
Pupils could construct and use a formula to transform one list of data to another.

Calculator methods

k use a calculator for calculations involving several digits, including decimals; use a calculator to solve number problems [for example, 4 □ × 7 = 343]; know how to enter and interpret money calculations and fractions; know how to select the correct key sequence for calculations with more than one operation [for example, 56 × (87 – 48)].

Solving numerical problems

4 Pupils should be taught to:

a choose, use and combine any of the four number operations to solve word problems involving numbers in 'real life', money or measures of length, mass, capacity or time, then perimeter and area

b choose and use an appropriate way to calculate and explain their methods and reasoning

c estimate answers by approximating and checking that their results are reasonable by thinking about the context of the problem, and where necessary checking accuracy [for example, by using the inverse operation, by repeating the calculation in a different order]

d recognise, represent and interpret simple number relationships, constructing and using formulae in words then symbols [for example, $c = 15n$ is the cost, in pence, of n articles at 15p each]

e read and plot coordinates in the first quadrant, then in all four quadrants [for example, plot the vertices of a rectangle, or a graph of the multiples of 3].

Ma3 Shape, space and measures

Using and applying shape, space and measures

1 Pupils should be taught to:

Problem solving

a recognise the need for standard units of measurement
b select and use appropriate calculation skills to solve geometrical problems
c approach spatial problems flexibly, including trying alternative approaches to overcome difficulties
d use checking procedures to confirm that their results of geometrical problems are reasonable

Communicating

e organise work and record or represent it in a variety of ways when presenting solutions to geometrical problems
f use geometrical notation and symbols correctly
g present and interpret solutions to problems

Reasoning

h use mathematical reasoning to explain features of shape and space.

Understanding properties of shape

2 Pupils should be taught to:

a recognise right angles, perpendicular and parallel lines; know that angles are measured in degrees and that one whole turn is 360 degrees and angles at a point total 360 degrees, then recognise that angles at a point on a straight line total 180 degrees; know that the sum of the angles of a triangle is 180 degrees
b visualise and describe 2-D and 3-D shapes and the way they behave, making more precise use of geometrical language, especially that of triangles, quadrilaterals, and prisms and pyramids of various kinds; recognise when shapes are identical
c make and draw with increasing accuracy 2-D and 3-D shapes and patterns; recognise reflective symmetry in regular polygons; recognise their geometrical features and properties including angles, faces, pairs of parallel lines and symmetry, and use these to classify shapes and solve problems
d visualise 3-D shapes from 2-D drawings.

Understanding properties of position and movement

3 Pupils should be taught to:

a visualise and describe movements using appropriate language
b transform objects in practical situations; transform images using ICT; visualise and predict the position of a shape following a rotation, reflection or translation

1c → ICT opportunity
Pupils could use software to create repeating patterns, such as tessellations.

1e, 1g → link to other subjects
These requirements build on En1/1a, 1c and En3/1a, 1e.

2c → ICT opportunity
Pupils could use object drawing software to plan alternative layouts for a room.

Note for 4a
In the international system of units, kilogram (kg) is the unit of mass. In practice, mass is measured by weighing; scales measure or compare a force (a push or a pull). Initially it is acceptable to treat weight as synonymous with mass but later in key stage 2 pupils will learn that the unit of weight (as type of force) is the newton.

c identify and draw 2-D shapes in different orientations on grids; locate and draw shapes using coordinates in the first quadrant, then in all four quadrants [for example, use coordinates to locate position in a computer game].

Understanding measures

4 Pupils should be taught to:

a recognise the need for standard units of length, mass and capacity, choose which ones are suitable for a task, and use them to make sensible estimates in everyday situations; convert one metric unit to another [for example, convert 3.17kg to 3170g]; know the rough metric equivalents of imperial units still in daily use

b recognise that measurement is approximate; choose and use suitable measuring instruments for a task; interpret numbers and read scales with increasing accuracy; record measurements using decimal notation

c recognise angles as greater or less than a right angle or half-turn, estimate their size and order them; measure and draw acute, obtuse and right angles to the nearest degree

d read the time from analogue and digital 12- and 24-hour clocks; use units of time – seconds, minutes, hours, days, weeks – and know the relationship between them

e find perimeters of simple shapes; find areas of rectangles using the formula, understanding its connection to counting squares and how it extends this approach; calculate the perimeter and area of shapes composed of rectangles.

Ma4 Handling data

Using and applying handling data

1 Pupils should be taught to:

Problem solving

a select and use handling data skills when solving problems in other areas of the curriculum, in particular science

b approach problems flexibly, including trying alternative approaches to overcome any difficulties

c identify the data necessary to solve a given problem

d select and use appropriate calculation skills to solve problems involving data

e check results and ensure that solutions are reasonable in the context of the problem

Communicating

f decide how best to organise and present findings

g use the precise mathematical language and vocabulary for handling data

Reasoning

h explain and justify their methods and reasoning.

Processing, representing and interpreting data

2 Pupils should be taught to:

a solve problems involving data

b interpret tables, lists and charts used in everyday life; construct and interpret frequency tables, including tables for grouped discrete data

c represent and interpret discrete data using graphs and diagrams, including pictograms, bar charts and line graphs, then interpret a wider range of graphs and diagrams, using ICT where appropriate

d know that mode is a measure of average and that range is a measure of spread, and to use both ideas to describe data sets

e recognise the difference between discrete and continuous data

f draw conclusions from statistics and graphs and recognise when information is presented in a misleading way; explore doubt and certainty and develop an understanding of probability through classroom situations; discuss events using a vocabulary that includes the words 'equally likely', 'fair', 'unfair', 'certain'.

1f → links to other subjects
This requirement builds on En3/1a, 1e.

1g → links to other subjects
This requirement builds on En1/1a, 1c.

Breadth of study

1 During the key stage, pupils should be taught the **Knowledge, skills and understanding** through:

a activities that extend their understanding of the number system to include integers, fractions and decimals

b approximating and estimating more systematically in their work in mathematics

c using patterns and relationships to explore simple algebraic ideas

d applying their measuring skills in a range of contexts

e drawing inferences from data in practical activities, and recognising the difference between meaningful and misleading representations of data

f exploring and using a variety of resources and materials, including ICT

g activities in which pupils decide when the use of calculators is appropriate and then use them effectively

h using mathematics in their work in other subjects.

The attainment targets for mathematics

About the attainment targets

An attainment target sets out the 'knowledge, skills and understanding that pupils of different abilities and maturities are expected to have by the end of each key stage'[1]. Except in the case of citizenship[2], attainment targets consist of eight level descriptions of increasing difficulty, plus a description for exceptional performance above level 8. Each level description describes the types and range of performance that pupils working at that level should characteristically demonstrate.

The level descriptions provide the basis for making judgements about pupils' performance at the end of key stages 1, 2 and 3. At key stage 4, national qualifications are the main means of assessing attainment in mathematics.

Range of levels within which the great majority of pupils are expected to work		Expected attainment for the majority of pupils at the end of the key stage	
Key stage 1	**1–3**	at age 7	**2**
Key stage 2	**2–5**	at age 11	**4**
Key stage 3	**3–7**	at age 14	**5/6**[3]

Assessing attainment at the end of the key stage

In deciding on a pupil's level of attainment at the end of a key stage, teachers should judge which description best fits the pupil's performance. When doing so each description should be considered alongside descriptions for adjacent levels.

Arrangements for statutory assessment at the end of each key stage are set out in detail in QCA's annual booklets about assessment and reporting arrangements.

Examples in the level descriptions

The examples in grey type are not statutory.

[1] As defined by The Education Act 1996, section 353a.
[2] In citizenship, expected performance for the majority of pupils at the end of key stages 3 and 4 is set out in end of key stage descriptions.
[3] Including modern foreign languages.

Attainment target 1: using and applying mathematics

Teachers should expect attainment at a given level in this attainment target to be demonstrated through activities in which the mathematics from the other attainment targets is at, or very close to, the same level.

Level 1

Pupils use mathematics as an integral part of classroom activities. They represent their work with objects or pictures and discuss it. They recognise and use a simple pattern or relationship.

Level 2

Pupils select the mathematics they use in some classroom activities. They discuss their work using mathematical language and are beginning to represent it using symbols and simple diagrams. They explain why an answer is correct.

Level 3

Pupils try different approaches and find ways of overcoming difficulties that arise when they are solving problems. They are beginning to organise their work and check results. Pupils discuss their mathematical work and are beginning to explain their thinking. They use and interpret mathematical symbols and diagrams. Pupils show that they understand a general statement by finding particular examples that match it.

Level 4

Pupils are developing their own strategies for solving problems and are using these strategies both in working within mathematics and in applying mathematics to practical contexts. They present information and results in a clear and organised way. They search for a solution by trying out ideas of their own.

Level 5

In order to carry through tasks and solve mathematical problems, pupils identify and obtain necessary information. They check their results, considering whether these are sensible. Pupils show understanding of situations by describing them mathematically using symbols, words and diagrams. They draw simple conclusions of their own and give an explanation of their reasoning.

Level 6

Pupils carry through substantial tasks and solve quite complex problems by independently breaking them down into smaller, more manageable tasks. They interpret, discuss and synthesise information presented in a variety of mathematical forms. Pupils' writing explains and informs their use of diagrams. Pupils are beginning to give mathematical justifications.

Level 7

Starting from problems or contexts that have been presented to them, pupils progressively refine or extend the mathematics used to generate fuller solutions. They give a reason for their choice of mathematical presentation, explaining features they have selected. Pupils justify their generalisations, arguments or solutions, showing some insight into the mathematical structure of the problem. They appreciate the difference between mathematical explanation and experimental evidence.

Level 8

Pupils develop and follow alternative approaches. They reflect on their own lines of enquiry when exploring mathematical tasks; in doing so they introduce and use a range of mathematical techniques. Pupils convey mathematical or statistical meaning through precise and consistent use of symbols that is sustained throughout the work. They examine generalisations or solutions reached in an activity, commenting constructively on the reasoning and logic or the process employed, or the results obtained, and make further progress in the activity as a result.

Exceptional performance

Pupils give reasons for the choices they make when investigating within mathematics itself or when using mathematics to analyse tasks; these reasons explain why particular lines of enquiry or procedures are followed and others rejected. Pupils apply the mathematics they know in familiar and unfamiliar contexts. Pupils use mathematical language and symbols effectively in presenting a convincing reasoned argument. Their reports include mathematical justifications, explaining their solutions to problems involving a number of features or variables.

Attainment target 2: number and algebra

Level 1

Pupils count, order, add and subtract numbers when solving problems involving up to 10 objects. They read and write the numbers involved.

Level 2)

Pupils count sets of objects reliably, and use mental recall of addition and subtraction facts to 10. They begin to understand the place value of each digit in a number and use this to order numbers up to 100. They choose the appropriate operation when solving addition and subtraction problems. They use the knowledge that subtraction is the inverse of addition. They use mental calculation strategies to solve number problems involving money and measures. They recognise sequences of numbers, including odd and even numbers.

Level 3

Pupils show understanding of place value in numbers up to 1000 and use this to make approximations. They begin to use decimal notation and to recognise negative numbers, in contexts such as money and temperature. Pupils use mental recall of addition and subtraction facts to 20 in solving problems involving larger numbers. They add and subtract numbers with two digits mentally and numbers with three digits using written methods. They use mental recall of the 2, 3, 4, 5 and 10 multiplication tables and derive the associated division facts. They solve whole-number problems involving multiplication or division, including those that give rise to remainders. They use simple fractions that are several parts of a whole and recognise when two simple fractions are equivalent.

Level 4

Pupils use their understanding of place value to multiply and divide whole numbers by 10 or 100. In solving number problems, pupils use a range of mental methods of computation with the four operations, including mental recall of multiplication facts up to 10 × 10 and quick derivation of corresponding division facts. They use efficient written methods of addition and subtraction and of short multiplication and division. They add and subtract decimals to two places and order decimals to three places. In solving problems with or without a calculator, pupils check the reasonableness of their results by reference to their knowledge of the context or to the size of the numbers. They recognise approximate proportions of a whole and use simple fractions and percentages to describe these. Pupils recognise and describe number patterns, and relationships including multiple, factor and square. They begin to use simple formulae expressed in words. Pupils use and interpret coordinates in the first quadrant.

Level 5

Pupils use their understanding of place value to multiply and divide whole numbers and decimals by 10, 100 and 1000. They order, add and subtract negative numbers in context. They use all four operations with decimals to two places. They reduce a fraction to its simplest form by cancelling common factors and solve simple problems involving ratio and direct proportion. They calculate fractional or percentage parts of quantities and measurements, using a calculator where appropriate. Pupils understand and use an appropriate non-calculator method for solving problems that involve multiplying and dividing any three-digit number by any two-digit number. They check their solutions by applying inverse operations or estimating using approximations. They construct, express in symbolic form, and use simple formulae involving one or two operations. They use brackets appropriately. Pupils use and interpret coordinates in all four quadrants.

Level 6

Pupils order and approximate decimals when solving numerical problems and equations [for example, $x^3 + x = 20$], using trial-and-improvement methods. Pupils are aware of which number to consider as 100 per cent, or a whole, in problems involving comparisons, and use this to evaluate one number as a fraction or percentage of another. They understand and use the equivalences between fractions, decimals and percentages, and calculate using ratios in appropriate situations. They add and subtract fractions by writing them with a common denominator. When exploring number sequences, pupils find and describe in words the rule for the next term or nth term of a sequence where the rule is linear. They formulate and solve linear equations with whole-number coefficients. They represent mappings expressed algebraically, and use Cartesian coordinates for graphical representation interpreting general features.

Level 7

In making estimates, pupils round to one significant figure and multiply and divide mentally. They understand the effects of multiplying and dividing by numbers between 0 and 1. Pupils solve numerical problems involving multiplication and division with numbers of any size, using a calculator efficiently and appropriately. They understand and use proportional changes, calculating the result of any proportional change using only multiplicative methods. Pupils find and describe in symbols the next term or nth term of a sequence where the rule is quadratic; they multiply two expressions of the form $(x + n)$; they simplify the corresponding quadratic expressions. Pupils use algebraic and graphical methods to solve simultaneous linear equations in two variables. They solve simple inequalities.

Level 8

Pupils solve problems involving calculating with powers, roots and numbers expressed in standard form, checking for correct order of magnitude. They choose to use fractions or percentages to solve problems involving repeated proportional changes or the calculation of the original quantity given the result of a proportional change. They evaluate algebraic formulae, substituting fractions, decimals and negative numbers. They calculate one variable, given the others, in formulae such as $V = \pi r^2 h$. Pupils manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions. They know that $a^2 - b^2 = (a+b)(a-b)$. They solve inequalities in two variables. Pupils sketch and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations.

Exceptional performance

Pupils understand and use rational and irrational numbers. They determine the bounds of intervals. Pupils understand and use direct and inverse proportion. In simplifying algebraic expressions, they use rules of indices for negative and fractional values. In finding formulae that approximately connect data, pupils express general laws in symbolic form. They solve simultaneous equations in two variables where one equation is linear and the other is quadratic. They solve problems using intersections and gradients of graphs.

Attainment target 3: shape, space and measures

Level 1

When working with 2-D and 3-D shapes, pupils use everyday language to describe properties and positions. They measure and order objects using direct comparison, and order events.

Level 2

Pupils use mathematical names for common 3-D and 2-D shapes and describe their properties, including numbers of sides and corners. They distinguish between straight and turning movements, understand angle as a measurement of turn, and recognise right angles in turns. They begin to use everyday non-standard and standard units to measure length and mass.

Level 3

Pupils classify 3-D and 2-D shapes in various ways using mathematical properties such as reflective symmetry for 2-D shapes. They use non-standard units, standard metric units of length, capacity and mass, and standard units of time, in a range of contexts.

Level 4

Pupils make 3-D mathematical models by linking given faces or edges, draw common 2-D shapes in different orientations on grids. They reflect simple shapes in a mirror line. They choose and use appropriate units and instruments, interpreting, with appropriate accuracy, numbers on a range of measuring instruments. They find perimeters of simple shapes and find areas by counting squares.

Level 5

When constructing models and when drawing or using shapes, pupils measure and draw angles to the nearest degree, and use language associated with angle. Pupils know the angle sum of a triangle and that of angles at a point. They identify all the symmetries of 2-D shapes. They know the rough metric equivalents of imperial units still in daily use and convert one metric unit to another. They make sensible estimates of a range of measures in relation to everyday situations. Pupils understand and use the formula for the area of a rectangle.

Level 6

Pupils recognise and use common 2-D representations of 3-D objects. They know and use the properties of quadrilaterals in classifying different types of quadrilateral. They solve problems using angle and symmetry properties of polygons and angle properties of intersecting and parallel lines, and explain these properties. They devise instructions for a computer to generate and transform shapes and paths. They understand and use appropriate formulae for finding circumferences and areas of circles, areas of plane rectilinear figures and volumes of cuboids when solving problems. They enlarge shapes by a positive whole-number scale factor.

Level 7

Pupils understand and apply Pythagoras' theorem when solving problems in two dimensions. They calculate lengths, areas and volumes in plane shapes and right prisms. Pupils enlarge shapes by a fractional scale factor, and appreciate the similarity of the resulting shapes. They determine the locus of an object moving according to a rule. Pupils appreciate the imprecision of measurement and recognise that a measurement given to the nearest whole number may be inaccurate by up to one half in either direction. They understand and use compound measures, such as speed.

Level 8

Pupils understand and use congruence and mathematical similarity. They use sine, cosine and tangent in right-angled triangles when solving problems in two dimensions. They distinguish between formulae for perimeter, area and volume, by considering dimensions.

Exceptional performance

Pupils sketch the graphs of sine, cosine and tangent functions for any angle, and generate and interpret graphs based on these functions. Pupils use sine, cosine and tangent of angles of any size, and Pythagoras' theorem when solving problems in two and three dimensions. They use the conditions for congruent triangles in formal geometric proofs [for example, to prove that the base angles of an isosceles triangle are equal]. They calculate lengths of circular arcs and areas of sectors, and calculate the surface area of cylinders and volumes of cones and spheres. Pupils appreciate the continuous nature of scales that are used to make measurements.

Attainment target 4: handling data

The attainment target does not apply at key stage 1.

Level 1

Pupils sort objects and classify them, demonstrating the criterion they have used.

Level 2

Pupils sort objects and classify them using more than one criterion. When they have gathered information, pupils record results in simple lists, tables and block graphs, in order to communicate their findings.

Level 3

Pupils extract and interpret information presented in simple tables and lists. They construct bar charts and pictograms, where the symbol represents a group of units, to communicate information they have gathered, and they interpret information presented to them in these forms.

Level 4

Pupils collect discrete data and record them using a frequency table. They understand and use the mode and range to describe sets of data. They group data, where appropriate, in equal class intervals, represent collected data in frequency diagrams and interpret such diagrams. They construct and interpret simple line graphs.

Level 5

Pupils understand and use the mean of discrete data. They compare two simple distributions, using the range and one of the mode, median or mean. They interpret graphs and diagrams, including pie charts, and draw conclusions. They understand and use the probability scale from 0 to 1. Pupils find and justify probabilities, and approximations to these, by selecting and using methods based on equally likely outcomes and experimental evidence, as appropriate. They understand that different outcomes may result from repeating an experiment.

Level 6

Pupils collect and record continuous data, choosing appropriate equal class intervals over a sensible range to create frequency tables. They construct and interpret frequency diagrams. They construct pie charts. Pupils draw conclusions from scatter diagrams, and have a basic understanding of correlation. When dealing with a combination of two experiments, pupils identify all the outcomes, using diagrammatic, tabular or other forms of communication. In solving problems, they use their knowledge that the total probability of all the mutually exclusive outcomes of an experiment is 1.

Level 7

Pupils specify hypotheses and test them by designing and using appropriate methods that take account of variability or bias. They determine the modal class and estimate the mean, median and range of sets of grouped data, selecting the statistic most appropriate to their line of enquiry. They use measures of average and range, with associated frequency polygons, as appropriate, to compare distributions and make inferences. They draw a line of best fit on a scatter diagram, by inspection. Pupils understand relative frequency as an estimate of probability and use this to compare outcomes of experiments.

Level 8

Pupils interpret and construct cumulative frequency tables and diagrams, using the upper boundary of the class interval. They estimate the median and interquartile range and use these to compare distributions and make inferences. They understand how to calculate the probability of a compound event and use this in solving problems.

Exceptional performance

Pupils interpret and construct histograms. They understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn. They select and justify a sample and method to investigate a population. They recognise when and how to work with probabilities associated with independent mutually exclusive events.